Bidirectional 2nd And 3rd Order Intermodulation Distortion Products

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The usual mathematical model of intermodulation distortion takes the first few terms of the Maclaurin series expansion of the transfer function,

$$V_{out} = V_0 + k_1 V_{in} + k_2(V_{in})^2 + k_3(V_{in})^3,$$

assumes an input signal to the DUT of the form

$$V_{in} = E_1 \cos(\omega_1 t) + E_2 \cos(\omega_2 t),$$

expands the input signal applied to the transfer function, and after application of trig identities and rearrangement of terms, develops the following output function:

$$V_{out} = V_0 + 1/2\, k_2(E_1^2 + E_2^2)$$

$$+ (k_1 E_1 + 3/4\, k_3 E_1^3 + 3/2\, k_3 E_1 E_2^2) \cos(\omega_1 t)$$

$$+ (k_1 E_2 + 3/4\, k_3 E_2^3 + 3/2\, k_3 E_1^2 E_2) \cos(\omega_2 t)$$

$$+ 1/2\, k_2 E_1^2 \cos(2\omega_1 t)$$

$$+ 1/2\, k_2 E_2^2 \cos(2\omega_2 t)$$

$$+ k_2 E_1 E_2 \cos((\omega_1 + \omega_2)t)$$

$$+ k_2 E_1 E_2 \cos((\omega_1 - \omega_2)t)$$

$$+ 1/4\, k_3 E_1^3 \cos(3\omega_1 t)$$

$$+ 1/4\, k_3 E_2^3 \cos(3\omega_2 t)$$

$$+ 3/4\, k_3 E_1^2 E_2 \cos((2\omega_1 + \omega_2)t)$$

$$+ 3/4\, k_3 E_1^2 E_2 \cos((2\omega_1 - \omega_2)t)$$

$$+ 3/4\, k_3 E_1 E_2^2 \cos((2\omega_2 + \omega_1)t)$$

$$+ 3/4\, k_3 E_1 E_2^2 \cos((2\omega_2 - \omega_1)t).$$

A variant of the above formula was given in “Don’t guess the spurious level of an amplifier. The intercept method gives the exact values with the aid of a simple nomograph,” by F. McVay, ***Electronic Design*** 3, February 1, 1967, 70 – 73.

But when signals enter a passive or active device at both the input and the output of the device, such as for an amplifier in each leg of a signal generator two tone IMD test setup, the single variable Maclaurin series transfer function model no longer applies because the transfer function is a function of two variables, namely the input and the output. This is also the case for transfer functions with two inputs which are not independent of frequency, such as wide spaced two tone IMD where a filter follows the combiner. In such cases the transfer function must be regarded as a Taylor series of two variables

$$V_{out}(V_1,V_2) = V_0 + k_{11}V_1 + k_{21}V_2 + k_{12}V_1^2 + c_{21}V_1V_2 + k_{22}V_2^2 + k_{13}V_1^3$$

$$+ c_{31}V_1^2 V_2 + c_{32}V_1^2V_2 + k_{23}V_2^3 + \ldots$$

and the two applied tones

$$V_1 = E_1 \text{ COS}(\omega_1 t) \text{ and } V_2 = E_2 \text{ COS}(\omega_2 t)$$

must be substituted for the independent variables V_1 and V_2 in the Taylor series to develop a correct model for the general two variable case.

After the usual expansion, application of trig identities, and collection of terms, the following output function is obtained:

$$V_{out}(V_1,V_2) = V_0 + 1/2\ k_{12}E_1^2 + 1/2\ k_{22}E_2^2$$

$$+ (k_{11}E_1 + 3/4\ k_{13}E_1^3 + 3/2\ c_{32}E_1E_2^2)\ \text{COS}(\omega_1 t)$$

$$+ (k_{21}E_2 + 3/4\ k_{23}E_2^3 + 3/2\ c_{31}E_1^2E_2)\ \text{COS}(\omega_2 t)$$

$$+ 1/2\ k_{12}E_1^2\ \text{COS}(2\omega_1 t)$$

$$+ 1/2\ k_{22}E_2^2\ \text{COS}(2\omega_2 t)$$

$$+ 1/2\ c_{21}E_1E_2\ \text{COS}((\omega_1 + \omega_2)t)$$

$$+ 1/2\ c_{21}E_1E_2\ \text{COS}((\omega_1 - \omega_2)t)$$

$$+ 1/4\ k_{13}E_1^3\ \text{COS}(3\omega_1 t)$$

$$+ 1/4\ k_{23}E_2^3\ \text{COS}(3\omega_2 t)$$

$$+ 1/4\ c_{31}E_1^2E_2\ \text{COS}((2\omega_1 + \omega_2)t)$$

$$+ 1/4\ c_{31}E_1^2E_2\ \text{COS}((2\omega_1 - \omega_2)t)$$

$$+ 1/4\ c_{32}E_1E_2^2\ \text{COS}((2\omega_2 + \omega_1)t)$$

$$+ 1/4\ c_{32}E_1E_2^2\ \text{COS}((2\omega_2 - \omega_1)t).$$

As can be seen from the expansion above, for the general two variable case IMD products occur at the same frequencies as for the single variable case. However, the relationships between and among some of the amplitudes of the two variable case are not necessarily the same as for the one variable case. For example, the amplitudes of the $3f_1$ and $3f_2$ two variable products need not be equal when E_1 and E_2 are equal, while they are equal for the single variable case. And for example, the amplitudes of the $2f_2$ +/- f_1 two variable products need not be equal to the amplitudes of the $2f_1$ +/- f_2 two variable products, while they are equal for the single variable case.

With a little algebra similar to that in the box at right the following can be derived.

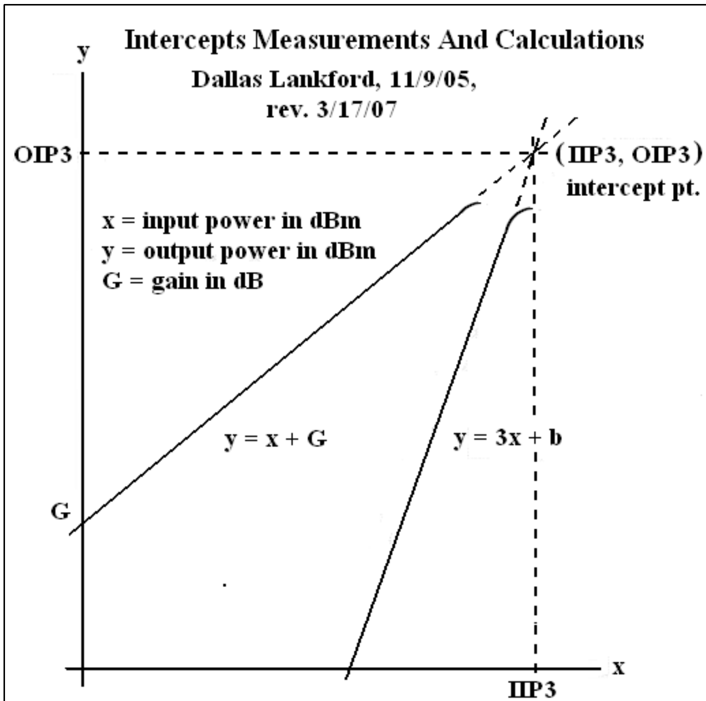
$y = 2x - IIP(f_1 + f_2) + G$

$y = 2x - IIP(f_1 - f_2) + G$

$y = 3x - 2IIP3(2f_1 \pm f_2) + G$

$y = 3x - 2IIP3(2f_1 \pm f_2) + G$

where G is the gain of the DUT, x is the input power of the two (equal) tones), y is the distortion output power, IIP denotes input intercept points, and the frequencies of the tones are as indicated.



From y = x + G we have OIP3 = IIP3 + G. And from y = 3x + b we have OIP3 = 3 IIP3 + b. Solving for b we get

b = OIP3 - 3 IIP3 = IIP3 + G - 3 IIP3 = G - 2 IIP3.

Substitution into the 2nd equation we find

y = 3x - 2IIP3 + G.

So it follows that IIP3 may be calculated from

IIP3 = (3p - (q - G))/2

if the (equal) input tones power p, the associated 3rd order intermodulation distortion output power q, and the device gain G are known.

Similarly, it can be shown that y = 2x - IIP2 + G and that

IIP2 = 2p - (q - G).